



# TONBRIDGE SCHOOL

Scholarship Examination 2010

**MATHEMATICS II**

Wednesday 28th April 2010  
2.00 pm

Time allowed: 1 hour 30 minutes

*Answer as many questions as you can.  
All the questions carry equal marks.*

*All answers must be supported by adequate explanation.  
Calculators may be used in any question.*

1. The base of a prism consists of a sector comprising a fraction  $f$  of a circle of radius  $r$  cm (so  $f = \frac{1}{2}$  gives a semicircle and  $f = \frac{1}{4}$  gives a quarter-circle). If the height of the prism is  $h$  cm, the total surface area  $S$  cm<sup>2</sup> of all of the faces of the prism is given by the formula  $S = 2f\pi r^2 + 2rh + 2f\pi rh$ , where  $\pi = 3.14159\dots$  has its usual meaning.

(a) If  $f = \frac{1}{3}$ ,  $r = 4.7$  and  $h = 5.8$  cm, find  $S$ .

(b) If  $S = 95$ ,  $f = 0.8$  and  $r = 2.3$  cm, find  $h$ .

(c) Factorise the expressions:

(i)  $2f\pi r^2 + 2rh + 2f\pi rh$ ,

(ii)  $2f\pi r^2 + 2f\pi rh$ .

2. (In this question, the answers are not necessarily whole numbers.)

(a) Solve the equations:

$$2x + \frac{1}{y} = 10.5,$$

$$x + \frac{2}{y} = 7.5.$$

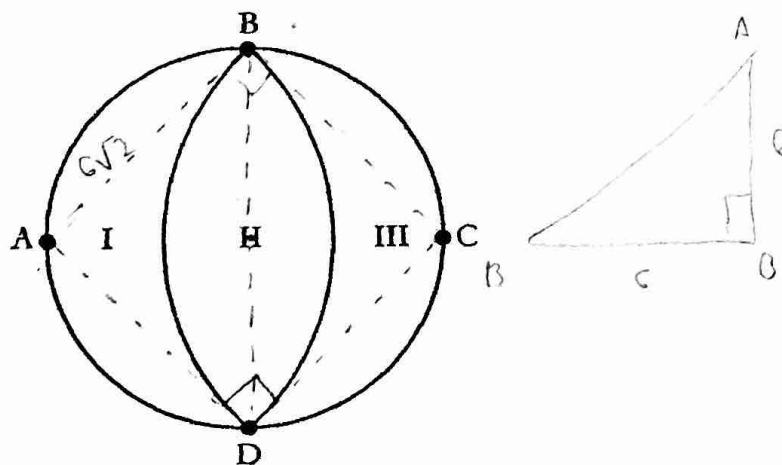
(b) Solve the equations:

$$ab^2 = 3,$$

$$a^2b = 72.$$

3. In the figure, A, B, C, D are the vertices (corners) of a square inscribed in a circle of radius 12 cm. Two arcs are drawn inside the circle: one has centre A and radius AB, the other has centre C and radius CD. These arcs divide the circle into three regions which are labelled I, II, III.

Area I =



By giving a suitable calculation, decide whether or not all three of the regions have the same area. (Hint: It may be helpful to draw the line BD.)

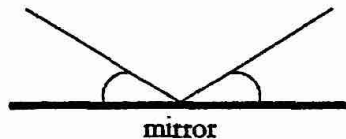
4. Fuel economy rates for cars are given in two forms:

- miles per gallon ("mpg"): the distance in miles gone when using one gallon of fuel;
- litres per 100 km ("lp100k"): the number of litres of fuel used in travelling 100 km.

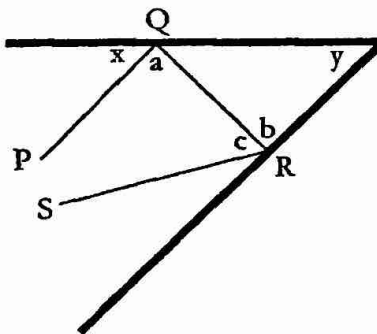
You are given that: 4.56 litres = 1 gallon and 0.62 miles = 1 km.

- What is 42.2 mpg in lp100k?
- What is 5.5 lp100k in mpg?
- Which uses more fuel on a 50 mile journey, a car with a fuel economy rate of 76 mpg or one with a rate of 3.9 lp100k?
- Find  $x$  if  $x$  mpg is the same fuel economy as  $x$  lp100k.

5. When light reflects off a mirror (as shown in the figure), the marked angles are equal.



The diagram below shows two mirrors and a ray of light reflecting off both of them with five angles  $a$ ,  $b$ ,  $c$ ,  $x$ ,  $y$  marked.



- Find, and simplify, expressions for:
  - $a$  in terms of  $x$ ,
  - $b$  in terms of  $x$  and  $y$ ,
  - $c$  in terms of  $x$  and  $y$ .
- Deduce that the rays PQ and RS are parallel for one (and only one) value of  $y$  which you should find.

6. This question concerns the graph whose equation is  $y = 6x^2 - x^3$ .
- By choosing sensible scales and calculating a sensible number of points, plot a neat graph of  $y = 6x^2 - x^3$  for values of  $x$  from 0 to 8.
  - What value of  $x$  gives the maximum value of  $y$  on your graph?
  - What would happen to the graph for negative values of  $x$ ?
  - A cube has side-length  $x$  cm. Its surface area numerically exceeds its volume by 20. Explain as clearly as you can why it follows that  $6x^2 - x^3 = 20$ .
  - Use your graph to find the value(s) of  $x$  between 0 and 8 that satisfy the equation in (d). Show clearly how you used your graph to find them.

7. This question is about the true statement:

*“If you double the sum of two square numbers, then the answer is also a sum of two square numbers.”*

For example:

$$2(1^2 + 2^2) = 10 = 1^2 + 3^2,$$

$$2(2^2 + 4^2) = 40 = 2^2 + 6^2,$$

$$2(3^2 + 7^2) = \dots = \dots + \dots,$$

$$2(5^2 + 6^2) = \dots = \dots + \dots,$$

$$2(8^2 + 13^2) = \dots = \dots + \dots.$$

- On your answer paper, complete the working for the three unfinished examples above.
- Study your answers and explain how you can predict the numbers on the right-hand side from those at the start on the left. That is, given  $2(x^2 + y^2) = A^2 + B^2$ , predict expressions for  $A$  and  $B$  in terms of  $x$  and  $y$ .
- Write as the sum of two square numbers: (i)  $2(45^2 + 46^2)$ , (ii)  $79^2 + 97^2 + 97^2 + 79^2$ .
- Find  $x$  and  $y$  with  $2(x^2 + y^2) = 123^2 + 321^2$ .